MATH 221 PRACTICE FINAL
Journal 3 and Journal 3.5

Our final exam will contain both concept questions and calculation questions. The concept questions will be similar to the concept questions from quizzes and at the end of each chapter of your journals. The calculation questions will be similar to: Calculation Practice pages in your journals, Calculation problems from each Chapter Review, and problems below.

These problems give an indication of the types of calculation problems you will see on the final exam. But remember that we only have 2 hours for our final, so I will pick and choose topics or add some details to particular problems to make sure time is not an issue.

1. Find the domain of $f(x, y) = \sqrt{1 - xy}$ and draw the contour map.

2. Find the $x = k$, $y = k$, and $z = k$ traces of $x^2 + y - 2z = 0$. Describe the surface.

3. Sketch the graph of the R^3 function $y^3 = z$.

4. Sketch the graph of $z = -x^2 - 2y^2$.

5. Compute the gradient ∇f for $f(x, y) = 3 + ln(5x - 2y)$, and then use this to give the equation of the line through $(1, 2)$ that is normal to the level curve at z=5, i.e., $3 + ln(5x - 2y) = 5$. What is the maximal directional derivative at the point $(1, 2)$?

6. For $f(x,y) = xe^{2y-x}$, compute the partial derivatives f_x, f_y, and all the second partial derivatives f_{xx}, f_{xy}, f_{yy}, f_{yx}.

7. Find the directional derivative of $g(x,y) = \sqrt{x^2 - y^2}$ at the point $(5,4)$ in the direction $< -\frac{5}{13}, \frac{12}{13} >$. Find the direction that maximizes the directional derivative.

8. Find the points (x,y) where $f(x,y) = 8x^3 - y^3 + 4xy$ has a local maximum, a local minimum, or a saddle point and clearly identify which is which.

9. Find the global extrema of $f(x,y) = x^2 - y^2$ on $x^2 + 4x + y^2 = 12$.

10. Find the extreme values of $f(x,y) = ye^x$ subject to the constraint $x^2 + 2y^2 = 2$.

11. Two students are working on the problem of maximizing and minimizing a function $f(x,y)$ on the region $R = \{(x,y) | x^2 + y^2 \leq 1\}$. They observe that $f(x,y)$ is continuous and differentiable everywhere in the xy-plane. The first student has determined that the maximum and minimum values of f on the circle $x^2 + y^2 = 1$ are 3 and -2. The second student has determined that there are only two solutions to $f_x(x,y) = 0$ and $f_y(x,y) = 0$, namely $(x,y) = (0, \frac{4}{3})$ and $(x,y) = (0, \frac{2}{3})$. Further they have determined that $f(0, \frac{2}{3}) = 7$ and $f(0, \frac{4}{3}) = -4$. Assuming their work is correct, what, if anything, can you say about the maximum and minimum values of $f(x,y)$ on R?

12. Find the dimensions of the closed rectangular box of maximum volume such that the sum of the length, width, and half of the height is 45.

13. Find the point on the plane $x - y + z = 4$ closest to the point $(1, 2, 3)$.

14. Evaluate $\int\int \frac{\sqrt{x}}{y^2} dA$ on the rectangle $0 \leq x \leq 1$ and $1 \leq y \leq 2$.

15. Evaluate $\int_0^\pi \int_x^\pi \frac{sin\, y}{y} dy\, dx$. Hint: reverse the order of integration.

16. Find the angle between $u =< 1, 0, 1 >$ and $v =< 1, 2, 2 >$.

17. Find the area of the parallelogram with sides $u =< 1, 0, 1 >$ and $v =< 1, 2, 2 >$.

18. Find the line through the points $(0, 1, 2)$ and $(1, 3, 4)$.

19. Find the plane through the points $(-1, 1, 1)$, $(0, 1, 2)$, and $(1, 3, 4)$.

20. For $r(t) =< \frac{1}{2}t^2, 2t, \frac{4}{3}t^{\frac{3}{2}} >$, find the velocity and speed functions.

21. Find the arc length of $r(t) =< \frac{1}{2}t^2, 2t, \frac{4}{3}t^{\frac{3}{2}} >$ for $0 \leq t \leq 1$.

22. Find the curvature of $r(t) =< \frac{1}{2}t^2, 2t, \frac{4}{3}t^{\frac{3}{2}} >$ at $t = 1$.

23. Find the tangent vector and the normal vector for $r(t) =< \frac{1}{2}t^2, 2t, \frac{4}{3}t^{\frac{3}{2}} >$ for $t = 1$.

24. Find the area from $r(t) =< \frac{1}{2}t^2, 2t, \frac{4}{3}t^{\frac{3}{2}} >$ to its projection in the xy-plane on $0 \leq t \leq 1$.

25. For $r(u, v) =< u\cos v, u\sin v, v >$ for $0 \leq u \leq 2, 0 \leq v \leq \frac{\pi}{2}$, find the tangent plane at the point when $u = 1$ and $v = \frac{\pi}{4}$.

26. Find the surface area of $r(u,v) =< u^2, uv, \frac{1}{2}v^2 >$ on $0 \leq u \leq 1, -1 \leq v \leq 1$.

27. Is $F(x,y) =< (4x+y^2)y, (2x+3y^2)x >$ a gradient field? If yes, find the potential function f.

28. Find the work integral of $F(x,y) =< (4x+y^2)y, (2x+3y^2)x >$ over $r(t) =< 1+t, 1+t^3 >$ on $0 \leq t \leq 2$.

29. Find the plane through the points $(6,0,3)$, $(5,1,4)$, and $(2,-1,3)$. Then use Lagrange multipliers to find the point on the plane closest to the origin.