MATH 221 PRACTICE MIDTERM
Journal 3, Chapters 1-3

Because we are not having a graded midterm in this class, you might wonder how well you are doing and how much you understand. Exams are a good way to assess your learning. Try the problems below, which are calculation problems taken from the exams of other 221 instructors in traditional in-person classes. If we were having a midterm, I'd use similar problems and add several concept questions, like the ones you see at the end of the journal chapters.

1. Find the domain of $f(x, y) = \sqrt{1 - x^2 - y^2}$ and identify the surface.

2. Find the $x = k$, $y = k$, and $z = k$ traces of $x^2 - y^2 + z^2 = 4$ and label each set of traces as either parabolas, hyperbolas, or ellipses.

3. Sketch the graph of $x^2 = z$.

4. Sketch the graph of $z = x^2 + \frac{1}{4}y^2$.

5. Compute the gradient ∇f for $f(x, y) = x + e^{xy}$, and then use this to give the equation of the line through $(2, 0)$ that is normal to the level curve $x + e^{xy} = 3$.

6. For $f(x,y) = x^2e^y + 2x - sin\, y$, compute the partial derivatives f_x, f_y, and all the second partial derivatives f_{xx}, f_{xy}, f_{yy}, f_{yx}.

7. Compute the gradient of $f(x,y,z) = xy + 2yz^2 + 3xyz$ and use it to compute the directional derivative in the direction $u =< 3/5, 0, 4/5 >$ at the point $(1, 0, -1)$.

8. Find the maximum directional derivative of $g(x,y) = 3 + ln(5x - 2y)$ at the point $(1,2)$ and the direction in which it occurs.

9. Find the points (x,y) where $f(x,y) = x^2 + y^4 + 4xy$ has a local maximum, a local minimum, or a saddle point and clearly identify which is which.

10. Find the absolute maximum and minimum of $f(x,y) = (x-1)^2 - y^2$ on the closed triangular region in the xy-plane with vertices $(-1,1)$, $(1,-1)$, and $(1,1)$.

11. Find the extreme values of $f(x,y) = ye^x$ subject to the constraint $x^2 + 2y^2 = 2$.

12. Find the dimensions of the rectangular box of maximum volume such that the sum of the length of its 12 edges is 60.