MATH 221 FINAL
Journal 3 and Journal 3.5

This is your chance to show off all that you have learned this semester. Before you start your exam, do the following–there's good science that these things improve test performance.

- Close your eyes and take 3 deep breaths.
- Stand up and strike a superhero pose for 10 seconds.

Instructions: You have three hours to complete this exam. OAKS will close access two hours after you begin. This exam has two parts. Part 1 consists of 29 concept questions worth one point each. Part 2 consists of 14 calculation questions worth five points each. You should be able to make your way through Part 1 in 10 to 20 minutes, leaving the remaining time for Part 2. Use paper to show all steps for each Part 2 problem, clearly numbering each problem. When you finish entering your multiple choice answers in OAKS, you must submit you pages of work in the Final Exam Dropbox within 10 minutes.

Now go ace this thing. You can do it!

Part 1: Concept Questions (1 point each)

1. A Pringles potato chip is a hyperbolic paraboloid defined over an elliptical region R.
   A. true
   B. false
2. Suppose the $x = k$, $y = k$, $z = k$ traces of $z = f(x, y)$ are all circles, then the surface must be
   A. ellipse
   B. sphere
   C. truncated sphere
   D. truncated ellipse
3. The planes $x + z = 4$ and $y = 5$ are perpendicular and intersect at the line $r(t) =< t, 5, 4 - t >$.
   A. true
   B. false
4. How many critical points does the surface created by your knuckle ridge have?
   A. 4
   B. 6
   C. 7
   D. 9
5. Suppose for some $z = f(x, y)$, $f_x(1, 4) = 0$ and $f_y(1, 4) = 3$, then the point $(1, 4)$ is
   A. definitely not a critical point.
   B. a critical point.
   C. a horizontal tangent plane.
   D. cannot be determined.

6. Which of the following is an $\Re^3$ surface with no critical points?

   A. $y = sin\, x$
   B. $z = cos\, x$
   C. $z = ln\, x$
   D. $z = x^2 + y^2$

7. $f_x(1, 6) = 3$ means

   A. when x increases one unit from $x = 1$ to $x = 2$, y held constant at $y = 6$, z increases about 3 units
   B. when x increases one unit from $x = 6$ to $x = 7$, y held constant at $y = 1$, z increases about 3 units
   C. when x increases one unit from $x = 1$ to $x = 2$, y held constant at $y = 6$, z decreases about 3 units
   D. when y increases one unit from $y = 6$ to $y = 7$, x held constant at $y = 6$, z increases about 3 units

8. Suppose for a particular function $z = f(x, y)$, the gradient is constant at every point on the surface, then the $\Re^3$ surface must be

   A. nonlinear.
   B. non-differentiable.
   C. local.
   D. linear.

9. A directional derivative evaluated at a point on a smooth continuous surface results in a

   A. scalar.
   B. vector.

10. Suppose (a, b) is a critical point and in the D-test $D(a, b) > 0$, then (a, b) is a global extrema.

    A. true
    B. false

11. Suppose $f(x, y)$ has a critical point at $(-1, 1)$ and $f_{xx}(-1, 1)$ and $f_{yy}(-1, 1)$ have opposite signs. Then the point $(-1, 1)$ must be a saddle point.

    A. true
    B. false

12. How many local extrema does $z = x^2 + y^2$ have? How many global extrema?

    A. 1, 2
    B. 1, infinitely many
    C. 1, none
    D. 1, 1

13. If $z = f(x, y)$ has a horizontal tangent plane at the point $(1, 7)$, then

    A. there is a local extrema at $(1, 7)$
    B. there is a global extrema at $(1, 7)$
    C. there is a saddle point at $(1, 7)$
    D. it cannot be determined; it could be A or B or C.

14. Suppose the global extrema of $z = f(x, y)$ over the region $x^2 + y^2 < 4$ are sought. Does the Extreme Value Theorem apply?

    A. yes

    B. no

15. Suppose the double integral of a function $z = f(x, y)$ evaluates to a negative value. Which of the following could be true?

    A. the function lies completely below the x-axis

    B. the function lies completely below the xy-plane

    C. the function lies partially below the xy-plane

    D. more of the function lies below the xy-plane than above

    E. A, B, C, D

    F. A, B, D

    G. B, D

16. If vectors $\mathbf{u}$ and $\mathbf{v}$ sum to $\mathbf{0}$, then the vectors have the same magnitude but point in opposite directions.

    A. true

    B. false

17. If vector $\mathbf{u}$ points in the northeast direction, then $a\mathbf{u}$ (where a is a scalar) also points in the northeast direction.

    A. true

    B. false

18. In $\Re^3$, two vectors either intersect or are parallel.

    A. true

    B. false

19. The line $r(t) =< 2t, 5t, 3 >$ is a horizontal line.

    A. true

    B. false

20. It is possible to compute the angle between two $\Re^8$ vectors.

    A. true

    B. false

21. When the curvature is 0 at every point t, then the function $r(t)$ is a line.

    A. true

    B. false

22. The equation $r(t) =< 1 + t, \frac{1}{t}, 3t >$ is a nonlinear space curve.

    A. true

    B. false

23. The curves $r(t) =< t, t^2 >$ for $0 \leq t \leq 4$ and $r(t) =< t^2, t^4 >$ for $0 \leq t \leq 2$ follow the same path but at different speeds.

    A. true

    B. false

24. The surface $r(u, v) =< u + v, 2u, \pi^2 u - v >$ is

    A. linear.

    B. nonlinear.

25. For the space curve $r(t)$, points at which $r'(t) =< x'(t), y'(t), 0 >$ are

    A. critical points.

    B. non-differentiable points.

    C. local extrema in height.

26. The gradient of a scalar function $f(x, y)$ is a vector field.

    A. true

    B. false

27. The flowlines of a gradient vector field move with the vectors in the field and perpendicular to the level curves of the contour map.

    A. true

    B. false

28. It is impossible to transform every $\Re^3$ vector field into a unit vector field.

    A. true

    B. false

29. If $F(x, y)$ is a conservative vector field, then $\frac{dM}{dx} = \frac{dN}{dy}$.

    A. true

    B. false

Part 2: Calculation Questions (5 points each)

30. Find the $y = k$ traces of $x^3 + y^2 = z$.

    A. $z = x^3 + k$
    B. $z = y^2 + k$
    C. $x^3 + y^2 = k$
    D. $z = \sqrt[3]{x^3 + k}$

31. Find the critical points of $z = 6xy - 2x^2y - 2xy^2$.

    A. (3, 0), (1, 3), and (0, 3)
    B. (3, 0), (1, 0), and (0, 0)
    C. (3, 0) and (1, 1)
    D. (0, 0), (3,0), (0, 3), and (1, 1)

32. Compute the directional derivative of $f(x, y) = \frac{x}{y}$ in the direction of $\mathbf{v} =< 1, 1 >$ at the point $(0, -1)$. Also find the direction that maximizes the directional derivative.

    A. $-\frac{1}{\sqrt{2}}; < 1, 0 >$
    B. $\frac{1}{\sqrt{2}}; < 1, 0 >$
    C. $\sqrt{2}; < -1, 1 >$
    D. $-\frac{1}{\sqrt{2}}; < -1, 0 >$

33. The point $(-\frac{7}{3}, \frac{1}{3})$ is a critical point of $f(x, y) = -x^3 - 5x^2 - 7x + y^3 + y^2 - y + 2$. This critical point is a

    A. local max.
    B. local min.
    C. saddle point.

34. Find the extreme values of $z = x^2 + xy$ on the region $R : y = 2x + 4$ for $-1 \leq x \leq 2$.

    A. global min at $(-\frac{2}{3}, \frac{8}{3}, -\frac{4}{3})$ and global max at $(2, 8, 20)$
    B. global min at $(-1, 2, -1)$ and global max at $(2, 8, 20)$
    C. global min at $(0, 4, 0)$ and global max at $(2, 8, 20)$
    D. global min at $(-\frac{2}{3}, \frac{8}{3}, -\frac{4}{3})$ and global max at $(2, 8, 10)$

35. Find the plane through the points $(6, 0, 3)$, $(5, 1, 4)$, and $(2, -1, 3)$. Then use Lagrange multipliers to find the point on this plane closest to the origin.

    A. $x - 4y + 5z = 21$; $(\frac{1}{2}, -2, \frac{5}{2})$
    B. $x + 4y + 5z = 21$; $(1, 2, 5.6)$
    C. $x - 4y - 5z = 21$; $(0, -2, 2.6)$
    D. $x - 4y + 5z = 21$; $(-2, 1, \frac{5}{2})$

36. Evaluate: $\int_0^1 \int_{2x}^2 e^{y^2}\, dydx$

    A. $e^4 - 14$
    B. 13.2876
    C. $\frac{1}{4}(e^4 - 1)$
    D. $e^4 - 1$

37. Find the line through the $\Re^4$ points $(0, 1, 2, 3)$ and $(1, 1, 3, 4)$.

    A. $r(t) =< 0, 1, 2, 3 > +t < 1, 0, 1, 1 >$ for $0 \leq t \leq \infty$
    B. $r(t) =< 0, 1, 2, 3 > +t < -1, 0, -1, -1 >$ for $-\infty \leq t \leq \infty$
    C. $r(t) =< 0, 1, 2, 3 > +t < 1, 0, 1, 1 >$ for $-\infty \leq t \leq \infty$
    D. $r(t) =< 1, 1, 3, 4 > +t < -1, 0, -1, -1 >$ for $0 \leq t \leq 1$

38. Find the curvature of $r(t) =< t^2, 2t, 3t >$ at $t = 1$.

    A. $\frac{\sqrt{52}}{17^{3/2}}$
    B. $\frac{\sqrt{52}}{\sqrt{17}}$
    C. 0
    D. $\frac{1}{10}$

39. Find the tangent and normal vectors of $r(t) =< t^2, 2t, 3t >$ at $t = 1$. Calculate the tangent vector by hand. Model and set up the normal vector, then use a tech tool to compute the normal vector.

    A. $\frac{1}{\sqrt{19}} < 2t, 2, 3 >; < .87, -.27, -.40 >$
    B. $\frac{1}{\sqrt{19}} < 2, 2, 3 >; < .87, -.27, -.40 >$
    C. $\frac{1}{\sqrt{17}} < 2, 2, 3 >; < .87, -.27, -.40 >$

40. Find the area from $r(t) =< t^2, t, ln\, t >$ to the xy-plane from $1 \leq t \leq 3$. Model and set up the integral, then use a tech tool to complete the integration.

    A. 8.268
    B. 6.036
    C. 5.084

41. Model the integral needed to compute the surface area of the Gaudi surface $r(u, v) =< u, v, .2u\, sin(\frac{v}{3}) >$ over the region $0 \leq u \leq 2, 1 \leq v \leq 6$. Then use tech tool to do the computation.

    A. 10.0462
    B. 12.0554
    C. 4.0185
    D. 10.1466

42. Is the vector field $F(x, y) =< 2x + ln\, y, 1 + \frac{x}{y} >$ conservative? Either way, compute the work done by the field to move a particle along the line segment from (1, 3) to (3, 4).

    A. yes; $8 + 3ln(4) - ln(3)$
    B. no; $8 + 3ln(4) - ln(3)$
    C. yes; $8 - 3ln(4) + ln(3)$
    D. no; $8 - 3ln(4) + ln(3)$

43. Is $r(t) =< 2e^{-t}, e^t >$ a flowline for $F(x, y) =< -x, y >$.

    A. yes
    B. no
    C. cannot be determined

Honor Pledge

44. I know what constitutes cheating on this exam and I did not cheat.

    A. true
    B. false