

MATH 105 EXAM #3

First, put your name on each page. Show all your work for partial credit. Please circle your answers. Good luck.

1.
 - (a) How many vertical tangent planes does the function $z = 2x^2 + 9y^2$ have?
 - (b) Where are they located?
 - (c) How many horizontal tangent planes does this surface have?
 - (d) Where are they located?
 - (e) Is every critical point for this surface an extrema?
2. List three things a partial derivative is useful for doing.
3.
 - (a) T or F. To ascend from the point $(1, 2)$ on a 3D surface the quickest possible way, continue to walk in the direction given by $\nabla f(1, 2)$.
 - (b) T or F. If the gradient at some point is $\langle 0, 0 \rangle$, then that point must be a critical point.
 - (c) T or F. The gradient gives the direction of steepest descent.
4. Which of the following is the best translation for the following statement: $f_x(3, 4) = 12$ for $z = f(x, y)$?
 - (a) a 1-unit change in x from 3 to 4 and y held constant at $y = 4$ results in a 12-unit increase in z
 - (b) a 1-unit change in y from 4 to 5 and x held constant at $x = 3$ results in a 12-unit increase in z
 - (c) a 1-unit change in x from 3 to 4 and a 1-unit increase in y from 3 to 4 results in a 12-unit decrease in z
 - (d) a 1-unit change in x from 3 to 4 and a 1-unit increase in y from 4 to 5 results in a 12-unit increase in z
 - (e) a 1-unit change in x from 3 to 4 and y held constant at $y = 4$ results in a 12-unit decrease in z
 - (f) a 1-unit change in y from 4 to 5 and x held constant at $x = 3$ results in a 12-unit decrease in z
 - (g) None of the above

5. Compute f_{xy} and f_{yy} for $z = f(x, y) = x^2y + e^{x-y}$.
6. Find the direction of steepest descent for the same function $z = f(x, y) = x^2y + e^{x-y}$ at the point $(2, 1)$. So to descend the surface from the point $(2, 1)$, which direction should you walk in (e.g., ENE, SW, ...)? Forever or just locally?
7. Draw the contour map for $z = f(x, y) = \ln(y + 2x^2)$. Describe in words and/or draw the \mathbb{R}^3 surface created from this contour map.
8. For $f(x, y) = 2x^2 - 2xy - 20x + 2y^2 - 2y + 5$, determine which of the following are CPs of $f(x, y)$.
- (a) (4,4)
 - (b) (7,7)
 - (c) (4,7)
 - (d) (7,4)
9. Use the contour map that you drew for $z_1 = f(x, y)$ in #7 above. Draw the contour map for the related \mathbb{R}^3 function $z_2 = f(x, y) - 3$. Be precise. Then answer the following questions:
- (a) Do the level curves move in the xy-plane? If so, how? If not, why not?
 - (b) Do the labels/values on the level curves change? If so, how? If not, why not?