Calculation Review

1. Compute the *local extrema* for each function.

(a) $f(x)=\frac{1}{3}x^3+2x^2+3x$

(b) $y=7-30x^2-\frac{8}{3}x^3+x^4$

(c) $y=(2x+5)/4$

(d) $f(t)=\frac{t+1}{2t^2+t+7}$

(e) $g(x)=\sqrt{x^2+1}$

(f) $f(x)=3x^4-8x^3-6x^2+24x$

(g) $y=x(x-3)^3$

(h) $y=\frac{x}{x^2+4}$

(i) $g(x)=-2x^3+6x^2+10$

(j) $y=2x^4-\frac{1}{3}x^6$

(k) $f(x)=xe^{-x}$

(l) $y=3x^{\frac{1}{3}}-x$

(m) $f(t)=\frac{t^2}{t^2+1}$

(n) $y=x+\sqrt{2-x}$

(o) $y=e^{-2x^2-x}$

(p) $f(x)=\ln(x^2+4)$

(q) $f(x)=\sqrt{x}(x-3)$

(r) $y=x(x-5)^3$

2. Find the points of *maximum curvature* for each function. What happens to the curvature as $x\to\infty$?

(a) $y=x^3$

(b) $y=\sqrt{x}$

(c) $y=e^x$

(d) $y=\ln x$

3. Compute the *global extrema* for each function on the given interval.

(a) $y=4x^3-3x^2+9x+12$ on $[-2, 1]$

(b) $y=8x^3+81x^2-42x$ on $[-8, 2]$

(c) $y=x\ln x$ on $[.2, 2]$

(d) $f(t)=2t^{\frac{1}{3}}-t$ on $[-4, 4]$

(e) $y=\frac{1}{5}x^5-x^4$ on $[-4, 4]$

(f) $y=x^2e^{-x}$ on $[0, 5]$

(g) $y=2x+5\cos x$ on $[0, 2\pi]$

(h) $g(x)=x^2+1/x$ on $[1, 4]$

(i) $y=\frac{x+1}{x-1}$ on $[-1, 1]$

(j) $y=(x^2-9)^2$ on $[-1, 4]$

(k) $f(x)=x\sqrt{4-x^2}$ on $[1, 2]$

(l) $y=\frac{1}{5}(3x+4)$ on $[0, 5]$

Calculation Review

4. Solve each *optimization problem*.

   (a) Find two positive numbers whose sum is 20 and product is maximal.

   (b) Find a positive number for which the sum of the number squared and its reciprocal is as small as possible.

   (c) Find the equation of the line through the point (4, 5) that cuts off the least area from the first quadrant.

   (d) An arch is represented by the part of the parabola $y=9-x^2$ that lies above the x-axis. Find the dimensions of the rectangular window of largest area that can be constructed inside the arch.

   (e) A rancher wants to build a three-sided, rectangular fence on his property along a river (the side along the river does not require fencing). Fencing costs $10 per yard, and the rancher wants to fence in an area of 450 square yards at the minimal cost. Find the dimensions of the fence.

   (f) A make-shift shelter can be made from a tarp draped over a ladder that is resting up against a high wall. If the ladder is 13-feet long, how far should the base of the ladder be away from the wall to maximize the volume of such a shelter?

   (g) Given a 10-inch by 10-inch square piece of paper and cut equal-sized squares from the corners appropriately, you can fold up the ends to create an open box (try it). What size squares should be cut from the corners to maximize the volume of this open box?

   (h) A 440-yard track is the perimeter of a rectangular field with semicircles on each end. Find the dimensions of the rectangular field that maximize its area.

Answers

CONCEPT

1. F
2. T
3. T
4. F
5. F
6. yes, EPs
7. objective, constraint
8. examples in chapter
9. CPs, EPs
10. f continuous, interval closed
11. concave up at x=3, no conclusion at x=2
12. 1st deriv in most cases
13. F
14. T
15. T
16. T
17. F
18. maybe
19. no
20. [1, 2], [3, 10]
21. y=sin x

CALCULATION

1. (a) l.min at x=−1, l.max at x=−3
   (b) l.min at x=−3, x=5, l.max: x=0
   (c) no local extrema
   (d) l.min at x=−3, l.max at x=1
   (e) l.min at x=0
   (f) l.min at x=−1, x=2, l.max: x=1
   (g) l.min at x=3/4
   (h) l.min at x=−2, l.max at x=2
   (i) l.min at x=0, l.max at x=2
   (j) l.min at x=0, l.max: x=2, x=−2
   (k) l.max at x=1
   (l) l.max at x=1
   (m) l.min at x=0
   (n) l.max at x=7/4
   (o) l.max at x=−1/4
   (p) l.min at x=0
   (q) l.min at x=1
   (r) l.min at x=5/4

2. (a) x=$\pm\sqrt[4]{\frac{1}{27}}$; as x $\to \infty$, $\kappa \to 0$
   (b) x=2; as x $\to \infty$, $\kappa \to 0$
   (c) x=.5 ln(.5); as x $\to \infty$, $\kappa \to 0$
   (d) x=$\frac{1}{\sqrt{2}}$; as x $\to \infty$, $\kappa \to 0$

3. (a) g.min at x=−2, g.max at x=1
   (b) g.min at x=$\frac{1}{4}$, g.max at x=−7
   (c) g.min at x=$\frac{1}{e}$, g.max at x=2
   (d) g.min at x=4, g.max at x=$(\frac{2}{3})^{\frac{3}{2}}$
   (e) g.min at x=4,−4, g.max at x=0
   (f) g.min at x=0, g.max at x=2
   (g) g.min at x=π−arcsin(2/5), g.max at x=2π
   (h) g.min at x=1, g.max at x=4
   (i) no g.min, g.max at x=−1
   (j) g.min at x=3, g.max at x=0
   (k) g.min at x=2, g.max at x=$\sqrt{2}$
   (l) g.min at x=0, g.max at x=5

4. (a) 10, 10
   (b) $\frac{1}{\sqrt[3]{2}}$
   (c) y=$-\frac{5}{4}$x+10
   (d) 6 × 3.46
   (e) 15 × 30
   (f) $\sqrt{6.5} \times \sqrt{6.5}$
   (g) $\frac{5}{3} \times \frac{5}{3} \times \frac{20}{3}$
   (h) 70.03 × 110