Calculation Review

1. Evaluate the *third derivative* for each function at x=1.

(a) $y=e^x+\sin x$
(b) $y=e^x+\ln x$
(c) $f(x)=e^{2x}-1/x^2-7x+4$
(d) $f(x)=\frac{1}{(x+1)^2}$
(e) $y=\frac{1}{(x^2+1)^2}$
(f) $f(x)=\cos x$
(g) $f(x)=\frac{x}{(x^2+1)^2}$
(h) $f(x)=ax^2+bx+c$
(i) $f(a)=ax^2+bx+c$
(j) $f(x)=xe^x$
(k) $f(x)=\sin x \cos x$
(l) $y=\ln(4x-3)$
(m) $y=5x^2+4x^{-1}$
(n) $y=\sin(x)$
(o) $f(x)=e^x-x^e$
(p) $f(x)=(7x-2)^4$
(q) $y=\ln(5x)$
(r) $y=\sin(e^x)$

2. Find the *critical points* and *possible inflection points* for each function.

(a) $y=x^2+4x+4$
(b) $f(x)=ax^2+bx+c$
(c) $f(a)=ax^2+bx+c$
(d) $f(x)=xe^x$
(e) $f(x)=\frac{x}{(x^2+1)^2}$
(f) $f(x)=\sin x$
(g) $f(x)=e^{x^2+x}$
(h) $y=e^{4-x^2}$
(i) $y=x \ln x$
(j) $f(x)=x^6-48x^2-4$
(k) $f(x)=\frac{1}{12}x^2-\frac{1}{2}x$
(l) $y=(x-32)^6$
(m) $y=x^3-2x^2-x+2$
(n) $y=\frac{3x+1}{x}$
(o) $f(x)=-1/x$
(p) $f(x)=-x^4+2x^3+36x^2$
(q) $y=\ln(x+1)$
(r) $y=2x^2-x^{-1}$

3. Find the *instantaneous rate of change* at each CP and each PIP for each function in #2.

Calculation Review

4. For each function, find the *IPs* and where the function is *concave up and concave down.*

   (a) $f(x)=7$
   (b) $f(x)=\frac{x^2+3}{x^2}$
   (c) $f(x)=x^5-5x+3$
   (d) $y=x(x+1)^2$
   (e) $y=(x+1)^3$
   (f) $f(x)=\frac{1}{12}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2-1$
   (g) $f(x)=e^{x+ln(x)}$
   (h) $f(x)=x^2 \ln x$ on $(0,\infty)$
   (i) $y=\frac{x-1}{x^2+1}$
   (j) $y=e^x$
   (k) $f(x)=-1/x$
   (l) $f(x)=\sqrt{2x-1}$
   (m) $y=\ln(x-4)$
   (n) $y=\frac{x}{x^2+4}$
   (o) $y=3x^5-5x^3+7$
   (p) $f(x)=x-2\sin x$

5. Compute the *curvature* and radius of the *osculating circle* for each function at the three points.

   (a) $y=x^4$ at $x=-\frac{1}{2}$, $x=0$, $x=\frac{1}{2}$
   (b) $y=\cos x$ at $x=0$, $x=\pi/2$, $x=\pi$
   (c) $y=x^2\,e^x$ at $x=-2$, $x=-1$, $x=0$

Answers

CONCEPT

1. F
2. T
3. T
4. T
5. T
6. nondiff at x=0
7. F
8. F
9. T
10. T
11. T
12. y=k
13. y=sin x
14. $=e^x$, $y=x^2$
15. $yy=x^3$
16. $y=e^x$
17. A
18. B
19. inc, c. down
20. y=mx+b
21. y=mx+b, $y=e^x$

CALCULATION

1. (a) $e^1-\cos(1)$
   (b) e^1+2
   (c) $8e^2+24$
   (d) $-3/4$
   (e) $-3/2$
   (f) sin(1)
   (g) 3/2
   (h) 0
   (i) 0
   (j) 4e
   (k) $4(\sin(1))^2-4(\cos(1))^2$
   (l) 32
   (m) -24
   (n) $-\cos(1)$
   (o) $e-e(e-1)(e-2)$
   (p) 41160
   (q) 2
   (r) $e^x\cos(e^x)-3e^{2x}\sin(e^x)-e^{3x}\cos(e^x)$

2. (a) CP: x=−2; no PIPs
   (b) CP: $x=\frac{-b}{2a}$; no PIPs
   (c) no CPs, no PIPs
   (d) CP: x=−1; PIP: x=−2
   (e) CPs: $x=\pm\frac{1}{\sqrt{3}}$; PIPs: x=0,1,−1
   (f) CPs: $x=\frac{\pi}{2} \pm 2\pi k$ for k=0,1,2, . . .; PIPs: $x=\pi \pm \pi k$ for k=0,1,2, . . .
   (g) CP: $x=-\frac{1}{2}$
   (h) CP: x=0; PIPs: $x=\pm\frac{1}{\sqrt{2}}$
   (i) CP: $x=e^{-1}$; no PIPs
   (j) CP: x=0,2,−2; PIPs: $x=\pm\frac{2}{\sqrt[4]{5}}$
   (k) CP: x=3; no PIPs
   (l) CP: x=32; PIP: x=32
   (m) CP: $x=0,\frac{4}{3}$; PIP: $x=\frac{2}{3}$
   (n) no CPs; no PIPs
   (o) no CPs; no PIPs
   (p) CPs: $x=0,\pm\frac{3\pm\sqrt{297}}{4}$; PIPs: x=−2,3
   (q) no CPs; no PIPs
   (r) CP: $x=-\frac{1}{\sqrt[3]{4}}$; PIP: $x=\frac{1}{\sqrt[3]{2}}$

3. Because they are critical points, we know that the instantaneous rate of change at each CP is 0. The instantaneous rates at PIPs are calculated by evaluating f'(PIP) for each PIP found for each function in #2 above.

Answers

CALCULATION

4. (a) No IPs, no concavity
   (b) No IPs, no concavity
   (c) IP: x=0, C. Up on $(0, \infty)$, C. Down on $(-\infty, 0)$
   (d) IP: x=$-\frac{2}{3}$, C. Up on $(-\frac{2}{3}, \infty)$, C. Down on $(-\infty, -\frac{2}{3})$
   (e) IP: x=-1, C. Up on $(-1, \infty)$, C. Down on $(-\infty, -1)$
   (f) No IP, C. Up on $(-\infty, \infty)$
   (g) IP: x=-2, C. Up on $(-2, \infty)$, C. Down on $(-\infty, -2)$
   (h) IP: x=$e^{-1.5}$, C. Up on $(e^{-1.5}, \infty)$, C. Down on $(0, e^{-1.5})$
   (i) IPs: x=-1, x=2 $+\sqrt{3}$, x=2$-\sqrt{3}$, C Up on $(-1, 2-\sqrt{3})$ and $(2+\sqrt{3}, \infty)$
   (j) No IPs, no concavity
   (k) No IPs, no concavity
   (l) No IPs, no concavity
   (m) No IPs, no concavity
   (n) IPs: x=0,$\pm\sqrt{12}$, C. Up on $(-\sqrt{12}, 0)$ and $\sqrt{12}, \infty)$, C. Down on $(-\infty, -\sqrt{12})$ and $(0, \sqrt{12})$
   (o) Ps: x=0,$\pm\frac{1}{\sqrt{2}}$, C. Up on $(-\frac{1}{\sqrt{2}}, 0)$ and $\frac{1}{\sqrt{2}}, \infty)$, C. Down on $(-\infty, -\frac{1}{\sqrt{2}})$ and $(0, \frac{1}{\sqrt{2}})$
   (p) IPs: x=$\pi \pm \pi k$ for $k = 0, 1, 2, \ldots$, C. Up on $(0, \pi)$ and $(2\pi, 3\pi)$ and . . .

5. (a) -2.15, 0, 2.15; r=.466, r=∞, r=.466
   (b) 1, 0, 1; r=1, r=∞, r=1
   (c) 2/e^2, e^2/(1+e^2)$^{1.5}$, 2; r=.5e^2, r=e^{-2}(1+e^2)$^{3/2}$, r=.5